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Fourier Analysis for Quantitative Interpretation of Self-Potential Anomalies Caused by Horizontal Cylinder and Sphere

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Abstract. Spectral analysis approach using Fourier transform has been developed for the interpretation of SP anomalies due to horizontal cylinder and sphere like structures. Fourier amplitude and phase spectra related to the SP anomaly are derived and analyzed in order to adopt an interpretation procedure aiming at evaluating the geometric and physical parameters of these two studied structures. The validity of the new proposed method has been tested on synthetic examples, where it has been found a close agreement between assumed and computed values. A field example from the Ergani Copper district, Turkey has also been analyzed and interpreted by the proposed method, where an acceptable agreement has been noticed between the obtained results and other published results.

Key Words: Fourier Transform, Self-Potential Anomalies, Spectral Analysis.

Introduction

Self-potential (SP) method, which is one of the oldest geophysical methods, plays an important role in the exploration of metallic sulphides . The quantitative interpretation of the SP anomaly is carried out by approximating the causative source to a simple body of regular geometric shape (viz, sheet, cylinder, sphere ... etc.), applying one of the following methods:

1 – Using only a few characteristic points on the totality of the anomaly curve, the methods, which were belonging to this category, were originally developed by Yungul 1950, Paul 1965 and Bhattacharya and Roy 1981. The es-

sential disadvantage of this method was related to the fact that only a few points are used on the anomaly curve and hence the interpreted results are not reliable. Taking into account that in most of the studied cases, the data was contaminated by noise. However, these methods were considered to be fast and suitable for giving rough estimation.

2 – Using the curve matching technique Meiser, 1962, according to this technique, the field curve is compared to the album of pre-computed theoretical curves. This process is cumbersoming and the complexity of the method is very high when the variables are numerous and when the process deals with a large number of anomalies.

3 – Using the least square method, the final solution could be found starting from a supposed initial solution Abdelrahman, *et al.,* 1997, Abdelrahman and Sharafldin, 1997.

4 – Using nonlinear programming technique, recently Asfahani and Tlas 2002, have developed a new interesting method by which the convergence towards the optimal solution is rapidly attained.

In this paper, it is proposed to use spectral analysis approach, by which the data is analyzed and interpreted in the wavenumber domain through employing the concept of Fourier transform (amplitude and phase spectra).

In fact, in recent years, the Fourier transform has gained popularity to be used for solving several problems in applied geophysics. This research extends this spectral analysis technique to interpret the SP anomalies due to horizontal cylinder and sphere structures. The characteristics of Fourier amplitude and phase spectra related to these structures are investigated and analyzed, which enabled to evaluate the parameters of the two mentioned structures.

Theory

In mineral exploration, the most polarized structures can be approximated into four categories: the sphere, the horizontal cylinder, the vertical cylinder and the inclined sheet.

In this paper, efforts are concentrated on studying of the horizontal cylinder and sphere structures only; Fig 1. The general equation, describing, the self-potential anomaly produced by these structures is written in the following form Yungul 1950, Bhattacharya and Roy 1981.

$$V(x_i, z, \theta, p) = -k \frac{x_i \cos\theta + z \sin\theta}{(x_i^2 + z^2)^p} \qquad i = 1, \ldots, N \tag{1}$$

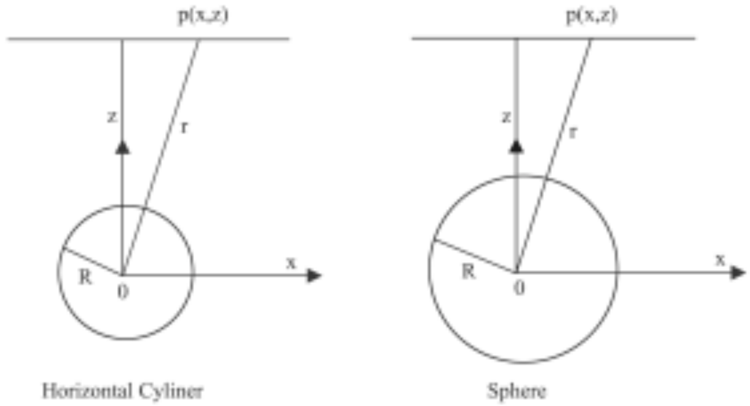

FIG. 1. Cross sectional view of a horizontal cylinder and a sphere.

where z : is the depth of the structure.
U : is the polarization angle.
x_i : is a position coordinate.
k : is the electrical dipole moment.
p : is related to the type of studied structure.

In this study, p is equal to 1 for a horizontal cylinder and to 1.5 for a sphere. Putting $x_i = 0$ in the equation (1), the following equation could be derived:

$$k = -\frac{V(0)z^{2p-1}}{\sin\ \theta} \tag{2}$$

where $V\,(0)$ is the anomaly value at the origin ($x_i = 0$).

Setting equation (1) to zero, the following equation is obtained:

$$\cot\ g\ \ \theta = -\frac{z}{x_0} \tag{3}$$

Substituting equation (2) and (3) in equation (1), it can be found:

$$V(x_i, z, p) = -\frac{V(0)z^{2p}(\frac{x_i}{x_0} - 1)}{(x_1^2 + z^2)^2} \tag{4}$$

In all the mathematical formulas described before, the knowledge of the axes of the self-potential have been assumed to determine V(0) and x_0. $V(0)$ could be evaluated by the intersect of the straight-line M1M2 with the anomaly profile (where M1 and M2 are the minimum and maximum values of $V(x)$).

Using equation (4), a typical SP anomaly profiles over:
1) Horizontal cylinder with $z = 2$ distance units, $\theta = 15$ degree and $k = 1000$ mV.
2) Sphere with $z = 2$ distance units, $\theta = 15$ degree, and k = 1000 mV.

Are computed for a profile length of 40 distance units and sampling interval of 1-unit as shown in Fig. 2a and Fig. 2b. The zero anomaly distance x_0 and the anomaly value at the origin $V(0)$ are also illustrated.

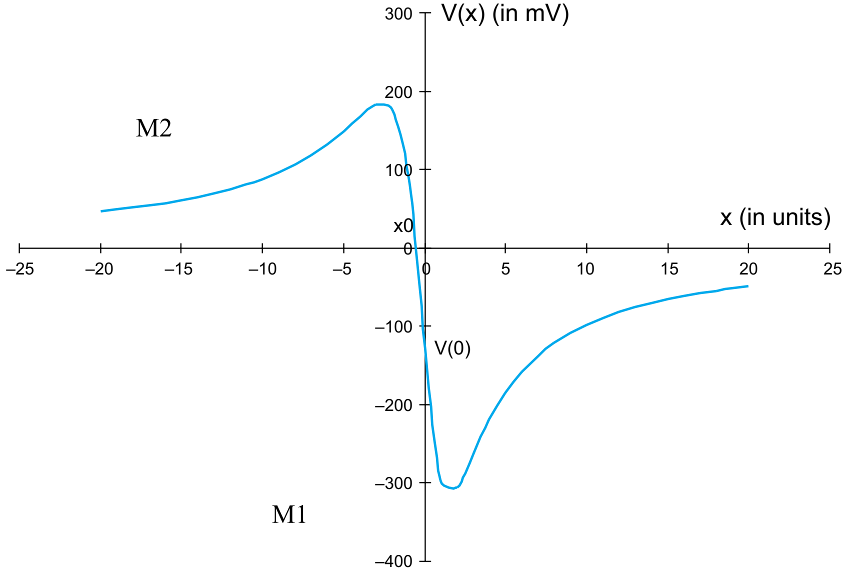

FIG. 2a. The SP anomaly response $V(x)$ over a horizontal cylinder obtained for $z = 2$ units, $\theta = 15$ degree and $k = 100$ mV.

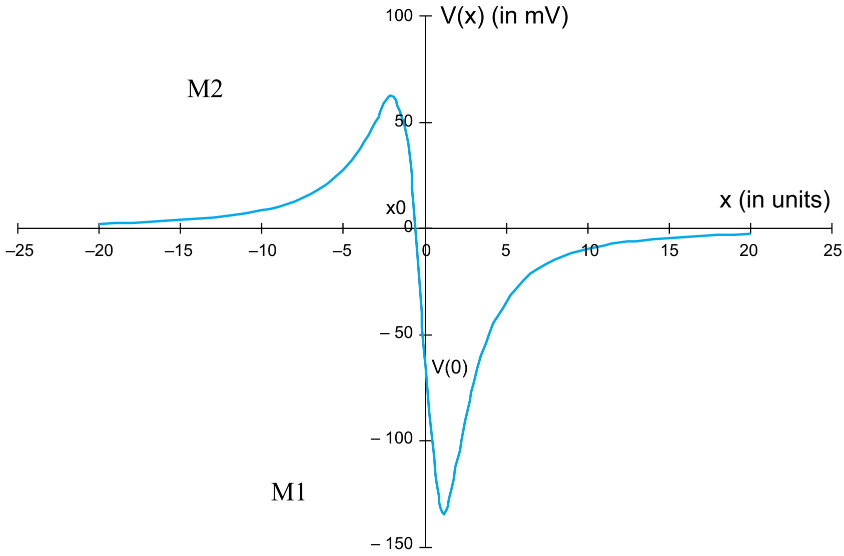

FIG. 2b. The SP anomaly response $S(x)$ over a sphere obtained for $z = 2$ units, $q = 15$ degree and $k = 1000$ mV.

Fourier Transformation

The estimation of the parameters (z, θ, k) in the case of horizontal cylinder and sphere like structure where p = 1 and 1.5 respectively is the goal of this research work by applying Fourier transformation. Therefore the characteristics of Fourier transformation are theoretically investigated and analyzed for p = 1 and p = 1.5.

1) Horizontal Cylinder (p = 1)

The Fourier transform $F(w)$ of the SP anomaly $V(x)$, in continuous form, is given by:

$$F(w) = \int_{-\infty}^{+\infty} V(x) e^{-iwx} dx \tag{5}$$

Replacing equation (4) in equation (5), it can be seen:

$$F(w) = \int_{-\infty}^{+\infty} -\frac{V(0)z^2}{x^2 + z^2} \left(\frac{x}{x_0} - 1\right) e^{-iwx} dx \tag{6}$$

The Fourier transform $F(w)$ is evaluated and obtained as:

$$F(w) = R(w) + i\, X(w) \tag{7}$$

Where $R(w)$ denotes the real part of $F(w)$ and is given by:

$$R(w) = \pi V(0)\ z\, e^{-wz} \tag{8}$$

And $X(x)$ denotes the imaginary part of $F(x)$ and is given by:

$$X(w) = \frac{\pi V(0) z^2}{x_0} e^{-wz} \tag{9}$$

The Fourier amplitude spectrum $A(w)$ and phase spectrum $\phi\ (w)$ are mathematically defined as:

$$A(w) = \sqrt{R^2(w) + X^2(w)} \tag{10}$$

And

$$\phi(w) = \frac{X(w)}{R(w)} \tag{11}$$

Substituting R(w) and $X(w)$ explained in equations (8) and (9), in equations (10) and (11), the following mathematical expressions are easily obtained

$$A(w) = \frac{\pi\, |V(0)|}{|x_0|} z \sqrt{x_0^2 + z^2}\ e^{-wz} \tag{12}$$

and $$\phi(w) = \frac{z}{x_0} \tag{13}$$

The analysis of the properties of both $A(w)$ and $\phi(w)$ in the equations (12) and (13) allows to evaluate the horizontal cylinder parameters (z, θ, k).

The Fourier amplitude spectrum A(w) in equation (12) is an exponentially decaying function.

Taking natural logarithms on both sides of this equation, the following equation is derived:

$$A1(w) = 1\text{n } A(w) = 1\text{n } \frac{\pi\,|V(0)|}{|x_0|} z\sqrt{x_0^2 + z^2} - w\,z \tag{14}$$

As known:

$$w_s = \frac{2\,\pi}{N} s \qquad\qquad s = 1,\ldots, N \tag{15}$$

Where N is the number of profile points. Substituting this equality in equation (14), it can be seen:

$$A1(s) = 1\text{n}\frac{\pi\,|V(0)|}{|x_0|} z\sqrt{x_0^2 + z^2} - \frac{2\pi}{N} z\,s \qquad s = 1,\ldots, N \tag{16}$$

Equation (16) indicates that, the slope of $A1(s)$ as a function of s is a straight line at higher frequencies, Fig. (3). The absolute value of this slope is denoted by m, this value allows determining the depth z of the horizontal cylinder as follows:

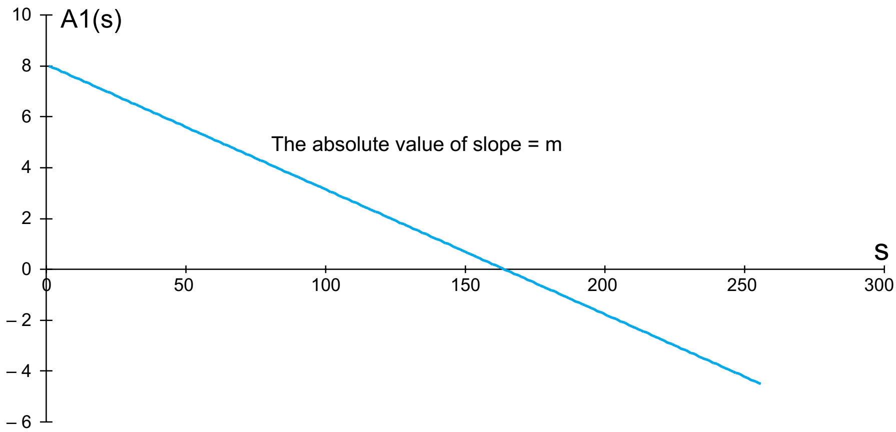

FIG. 3. Theoretical amplitude spectrum A$1(s)$ for the SP anomaly shown in Fig. 2a.

$$m = \frac{2\,\pi}{N}\; z \Rightarrow z = \frac{m\,N}{2\,\pi} \tag{17}$$

The analysis of phase spectrum $\phi(w)$ explained in equation (13), permits to evaluate the angle θ at higher frequencies by using the equation (3) discussed previously as follows:

$$\theta = 90 - arctg\,[-\phi(w)] \tag{18}$$

2 – Sphere (p = 1.5)

The theoretical Fourier transform in the sphere case where p = 1.5 is not easy to be analytically obtained. Bessel and modified Bessel functions are required in such a case, which complicates considerably the solution. Therefore the Fourier amplitude and phase spectra are evaluated by using discrete Fourier transform (*DFT*). The suitable *DFT* algorithm used in this research has the following form:

$$A(s) = \frac{1}{\sqrt{N}} \sum_{r=1}^{r=N} V(s) e^{\frac{2\pi i (r-1)(s-1)}{N}} \qquad s = 1,\ldots,N \tag{19}$$

and

$$A1(s) = 1\text{n}\, A(s) \qquad s = 1, \ldots, N \tag{20}$$

The absolute value of the slope of $A1(s)$ in such a case, Fig. 4 is given by the following empirical extrapolated equation:

$$m = \frac{2\pi}{N} \; z \Rightarrow z = \frac{m\,N}{2\pi} \tag{21}$$

Equation (21) gives a good approximation concerning the depth z of the sphere center as illustrated by the synthetic examples analyzed in the models 4,5 and 6 shown in Table 1.

TABLE 1. Synthetic examples k = 1000 mV, profile length = 40 distance units, sampling interval = 1 unit.

Model number	Assumed parameters			Evaluated parameters			Percentage of error in %		
	P	z	θ	z	θ	k	z	θ	k
1	1	2	15	2.0045	15.0267	1000.55	0.229	0.178	0.055
2	1	4	30	4.005	29.893	1004.51	0.1275	0.354	0.4506
3	1	6	75	5.981	75.35	995.253	0.313	0.467	0.4746
4	1.5	2	15	2.0324	16.966	915.9308	1.62	13.106	8.4069
5	1.5	4	30	4.013	30.963	978.1739	0.325	3.21	2.1826
6	1.5	6	75	6.022	75.963	1002.968	0.366	1.284	0.2968

The angle θ could be also evaluated by the same manner using equation (18) discussed previously.

Examples

I – Synthetic examples

Various cases including horizontal cylinder (p = 1) and sphere (p = 1.5) models are analyzed and tested as shown in Table 1. The discrete Fourier transform for each anomaly model is computed using the fast Fourier transform for a profile of length 40 distance units digitized at 1-distance unit intervals.

Fourier amplitude spectra $A1(s)$ have been computed for the model (1) presenting a horizontal cylinder and for the model (4) presenting a sphere structure as shown in Figs. 4 and 5 . It is to notice on these two figures that the logarithm Fourier amplitude spectrum $A1(w)$ is a straight line in the high frequencies region having the absolute value of slope (m) of 0.0492 and 0.0352 respectively. Depending on the equations (17) and (21), these obtained slopes allow determining the depth of horizontal cylinder and sphere structures as equal to 2.0045 units and 2.0324 units respectively.

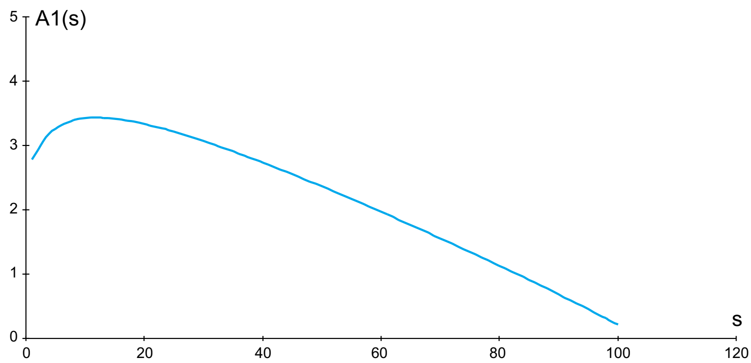

FIG. 4. Computed amplitude spectrum $A1(s)$, obtained by *DFT* for the SP anomaly shown in Fig. 2b.

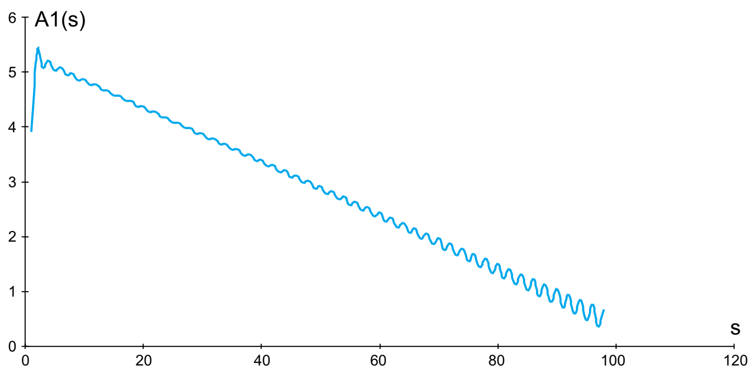

FIG. 5. Computed amplitude spectrum $A1(s)$, obtained by *DFT* for the SP anomaly shown in Fig. 2a.

The phase spectra $\phi(w)$ of the Fourier transform are computed for the same two models (1) and (4), as shown in Figs. 6 and 7. Depending on the equation (18), the polarization angle θ for the horizontal cylinder structure is obtained as equal to 15.0267 degree, whereas, this angle is equal 16.966 degree in the case of sphere presented in model (4).

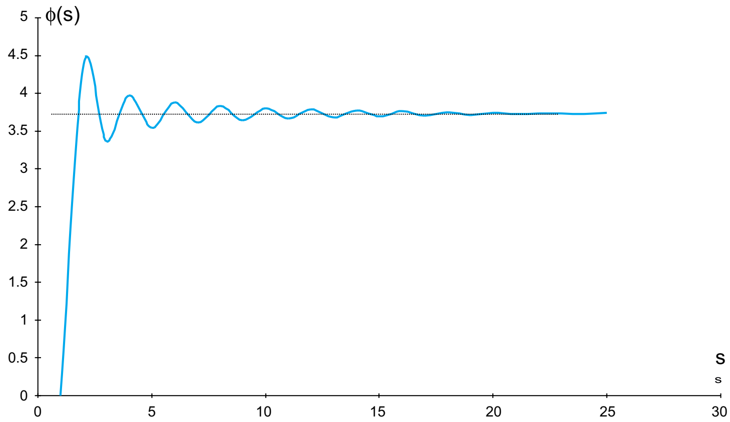

FIG. 6. Computed phase spectrum $f(s)$, obtained by *DFT* for the SP anomaly shown in Fig. 2a.

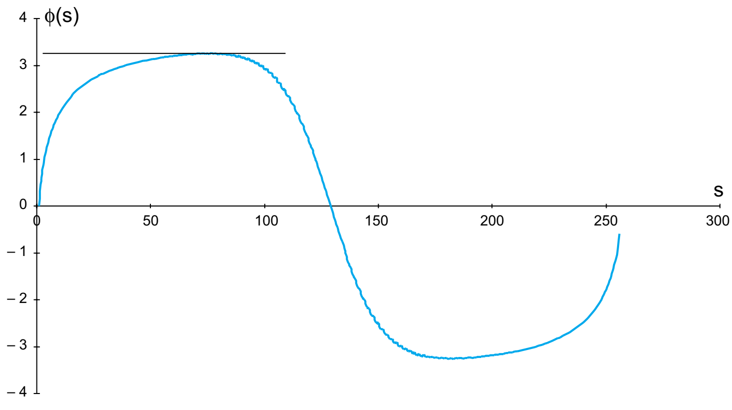

FIG. 7. Computed phase spectrum $\phi(s)$, obtained by *DFT* for the SP anomaly shown in Fig. 2a.

In all the studied cases (Table 1), it is evident the good agreement between the assumed and evaluated parameters. This agreement proves the validity and the efficiency of the spectral analysis proposed for the interpretation of SP anomalies.

II – Field Example

Fig. 8 shows the Suleymankoy self potential anomaly, Ergani copper district, 65 km SE of Elazig in eastern Turkey Bhattcharya and Roy, 1981. The SP measurements were performed and described in Yungul 1950. They represent the anomaly as a result of a polarized copper ore body. The SP anomaly has been digitized over a length of 262 meters at 1-meter intervals, and subjected to a spectral analysis using Fourier transformation. The interpretation of this anomaly is carried out in two cases, where $p = 1$ and $p = 1.5$. The logarithm Fourier amplitude spectrum $A1(s)$ of the anomaly curve is shown in Fig. 9. The absolute value of slope of this spectrum is equal to 0.0354 that allow determining the depth z corresponding to the horizontal cylinder and sphere like structures according to the equations (17) and (21).

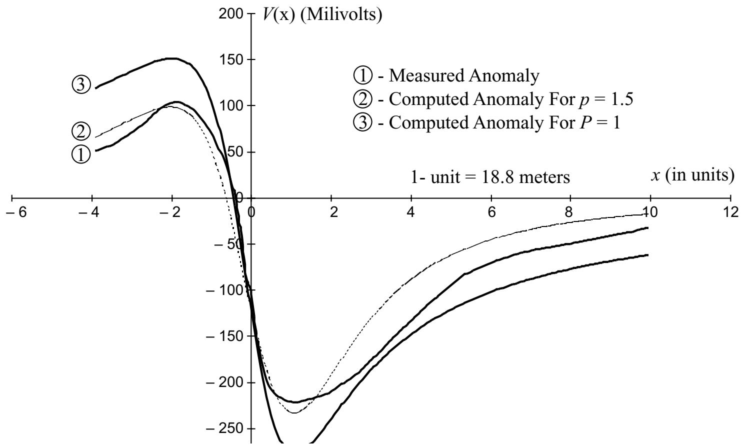

FIG. 8. SP anomaly over a polarized copper over body in Ergani-district, Turkey. The theoretical curves generated using the evaluated parameters are also shown.

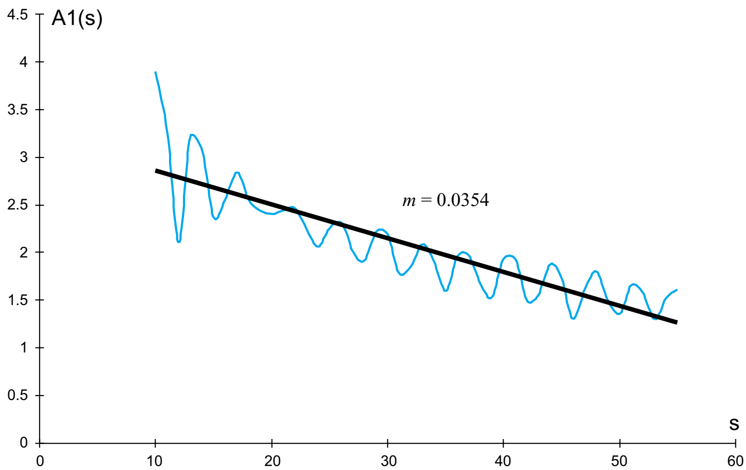

FIG. 9. Computed amplitude spectrum $A1(s)$ for the SP anomaly shown in Fig. 8.

The model parameters determined in the case of horizontal cylinder (p = 1) are z = 1.47785 units (1-unit = 18.8 m), θ = 17.25 degrees and k = 920.63 mV. The model parameters evaluated in the sphere case (p = 1.5) are z = 2.09 units (1-unit = 18.8 m), θ = 17.25 degrees and k = 1841.27 mV. The results of this interpretation using spectral analysis are presented and summarized in Table 2.

TABLE 2. Results of analysis of the SP anomaly over Ergani district, Turkey.

Evaluated parameters	P = 1	P = 1.5
Depth z in units	1.47785	2.09
Polarization angle θ in degree	17.25	17.25
Electric dipole moments in k in mV	920.63	1841.27
x_0 in units	– 0.45888	– 0. 64896

Using these evaluated parameters for p = 1 and p = 1.5, the theoretical profiles have also been generated for direct comparison as shown in Fig. 8.

It is obviously clear, that there exists an acceptable match between the observed and the computed anomalies for p = 1 and p = 1.5. The comparison between the two computed anomalies for p = 1 and p = 1.5 is carried out through studying of the following error function (Standard error):

$$SE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[L_i - V(x_i, z, \theta, k)]^2}$$

Where N is the number of observation points. L_i (i = 1, ... , N) and $V(x_i, z, \theta, k)$ are the observed and evaluated SP respectively. This computed error function are equal to 101.3 and 25.4 for p = 1 and p = 1.5 respectively. This indicates that the shape of the source resembles a sphere or practically a 3-D source with a hemispherical roof and depth to the center of 2.09 units (1-unit = 18.8 meters). The interpretation results obtained by the proposed method agree with those obtained by Yungal 1950, Bhattacharya 1981, Shalivahan *et al.* (1998), and Abdelrahman and Sharafeldin 1997 as shown in Table 3.

Summary and Conclusions

Spectral analysis approach using Fourier transform has been developed for the interpretation of SP anomalies due to horizontal cylinder and sphere like structures. Fourier amplitude and phase spectra of the Fourier transform have been given for the two studied structures. The proposed method is well validated with synthetic data, where a close agreement has been found between the assumed and evaluated parameters. The application of this method to a field data taken from Tureky resulted in good agreement between observed and computed anomalies for p = 1 and p = 1.5. The comparison between these two computed anomalies based on the studying of error function indicates that spherical model is more adapted to represent the observed SP anomaly. As a conclusion result, the proposed method gives reasonably good results, and an idea about the shape of the structure responsible of SP anomaly. Therefore, this method can be used for the interpretation of SP anomalies related to the horizontal cylinder and sphere like structures.

TABLE 3. Interpretation of SP profile (anomaly profile after Yungul, 1950). Comparison of results for sphere and horizontal cylinder like structures.

Model	Evaluated parameters	Yungul (1950)	Bhattacharya & Roy (1981)	Shalivahan *et al.* (1998)	Abdelrahman & Sharafeldin (1997)	Present study
Sphere	z in meters	53.8	54	51	42	39.3
	θ in degrees	26	30	30	13	17.25
	k in mV	–	–	–	2458	1841.27
	x_0 in meters	– 32.5	– 30	– 30	–9.4	– 12.2
Horizontal cylinder	z in meters	43	38.8	45	–	27.78
	θ in degrees	21	15	23	–	17.25
	k in mV	–	–	–	–	920.63
	x_0 in meters	–10	–16.7	–12	–	–8.63

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التفسير الكمي لشواذات الكمون الذاتي الناتجة عن اسطوانة أفقية وكرة باستخدام التحليل الطيفي وتحويلات فورييه

جمال أصفهاني، محمد طلاس و مصطفى حمادي

هيئـة الطاقـة الذريـة ، **دمشق** – *سورية*

المستخلص . طورنا في هذا البحث واقترحنا طريقة جديدة تعتمد على التحليل الطيفي وتحويلات فورييه لتفسير شواذات الكمون الذاتي الناتجة عن بنيات كروية واسطوانية أفقية. استنتجنا توابع السعة والطور الطيفية المتعلقة بشواذات الكمون الذاتي المدروسة من أجل صياغة منهجية تفسيرية لتقدير المحددات الفيزيائية والهندسية للبنيات المدروسة. اختبرنا صلاحية الطريقة الجديدة المقترحة على أمثلة صنعية ووجدنا توافقاً ممتازاً بين القيم المفروضة نظرياً والقيم المحسوبة. طبقنا هذه الطريقة التفسيرية على معطيات حقلية مأخوذة من تركيا ووجدنا أيضاً توافقاً مقبولاً بين نتائج هذه الطريقة ونتائج أخرى منشورة عالجت نفس الشاذ المدروس.